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THE STABILITY OF POISEUILLE FLUID OF A TWO-PHASE FLUID WITH A NONUNIFORM PARTICLE DISTRIBUTION

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The problem of the hydrodynamic stability of rarefied suspensions was formulated first in [1]. A two-phase medium consisting of a carrier fluid (gas) and dispersed solid particles suspended in it was considered. The following was assumed: the particles are influenced by a drag force proportional to the velocity; their volume concentration is small; and their distribution in the flow is uniform. Under these conditions, as was shown in [1], the stability problem amounts to solving the Orr–Sommerfeld equation with an effective complex velocity profile, and the Squire theorem is valid. Qualitative analysis in [1] indicates that fine dispersed particles suspended in a fluid destabilize its flow, whereas coarse ones stabilize it.

Systematic studies of two-phase Poiseuille flow stability conducted in [2–4] essentially augmented these findings. It was established that the value of stabilization was a monotone function of the dispersed-phase mass concentration. If the latter is high enough, then in a wide range of flow velocities infinitesimally small two-dimensional disturbances can be totally suppressed. Depending on particle size and density, this leads either to an increase in the critical Reynolds number (possibly by an order of 2–3 times) or to a separation of the instability domain into a couple of unconnected subdomains in which the flow is stable with respect to two-dimensional disturbances of any wavelength.

As a practical matter it is important to study the effect of dispersed-phase distribution inhomogeneity on the stability of two-phase flows. The solution of this problem is the goal of the present work. The problem of hydrodynamical stability is studied on the example of a Poiseuille flow of a strongly rarefied two-phase medium. The particles are assumed to be spherical and solid.

Statement of the Problem. The equations of the motion of the phases of a strongly rarefied two component fluid in dimensionless variables can be written as in [1] (the limits of applicability of this model were studied in [2]):

$$\nabla\cdot\mathbf{V}_f=0,\qquad \frac{\partial\rho_p}{\partial t}+\nabla\cdot(\rho_p\mathbf{V}_p)=0\,,$$
$$\frac{\partial\mathbf{V}_f}{\partial t}+\mathbf{V}_f\cdot\nabla\mathbf{V}_f=-\nabla P+\frac{1}{\mathrm{Re}}\Delta\mathbf{V}_f+\frac{\rho_p}{S\mathrm{Re}}(\mathbf{V}_p-\mathbf{V}_f)\,,\tag{1}$$
$$\frac{\partial\mathbf{V}_p}{\partial t}+\mathbf{V}_p\cdot\nabla\mathbf{V}_p=\frac{1}{S\mathrm{Re}}(\mathbf{V}_f-\mathbf{V}_p)\,,$$

where $\mathbf{V}_f$ and $\mathbf{V}_p$ are the velocity fields of the carrier and dispersed phases, respectively; P is the medium pressure; ρ_p is the mass concentration of the dispersed phase; $\rho_f=1$ is the density of the carrier medium; $\mathrm{Re}=U_0L/\nu$ is the Reynolds number; U_0 is the characteristic speed of the flow; L is the characteristic linear scale of the flow; ν is the viscosity of the carrier medium; the relaxation time S of the geterogenous medium is determined by a precise drag condition, in particular, if the Stoks low is valid, then $S=(2/9)(a/L)^2(\rho_p^*/\rho_f)$ (a and ρ_p^* are the particle size and density of its material).

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The problem of flow stability with respect to infinitesimally small disturbances is posed in a standard way. It was shown in [2–4] that the Squire theorem is valid for an arbitrary dispersed-phase distribution; therefore we can limit our consideration to the two-dimensional case:

$$\begin{pmatrix} \mathbf{V}_f \\ \mathbf{V}_p \\ \rho_p \end{pmatrix}(\mathbf{r},t) = \begin{pmatrix} \mathbf{U} \\ \mathbf{U} \\ f \end{pmatrix}(y) + \begin{pmatrix} \mathbf{v}_f \\ \mathbf{v}_p \\ f_1 \end{pmatrix}(\mathbf{r},t) = \begin{pmatrix} \mathbf{U} \\ \mathbf{U} \\ f \end{pmatrix}(y) + \begin{pmatrix} \mathbf{u}_f \\ \mathbf{u}_p \\ \varphi \end{pmatrix}(y)\, e^{i(\alpha x - \omega t)}, \tag{2}$$

$$P(\mathbf{r},t) = p(x) + p_1(\mathbf{r},t) = p(x) + \pi(y)e^{i(\alpha x - \omega t)}.$$

Here $\mathbf{U} = (U(y), 0, 0)$ is the flow velocity profile; α and ω are the wave number and disturbance frequency; and f is the distribution of the dispersed phase particles in the undisturbed flow. The disturbance amplitude is assumed to be small:

$$v_f \ll U, \quad v_p \ll U, \quad f_1 \ll f, \quad p_1 \ll p.$$

In this case the motion of the particles is determined by the motion of the carrier medium:

$$u_{px} = \frac{1}{1 + i\alpha S\mathrm{Re}(U-c)}\left(u_{fx} - \frac{S\mathrm{Re}}{1 + i\alpha S\mathrm{Re}(U-c)}\frac{dU}{dy}u_{fy}\right), \quad u_{py} = \frac{1}{1 + i\alpha S\mathrm{Re}(U-c)}u_{fy}, \tag{3}$$

whereas system (1) is reduced to the equation [2, 4]

$$(W - c)\Delta\psi - W''\psi + \frac{d}{dy}(\psi J f') = \frac{1}{i\alpha\mathrm{Re}}\Delta^2\psi, \tag{4}$$

where

$$W(y) = U + fJ; \qquad J = \frac{U - c}{1 + i\alpha S\mathrm{Re}(U-c)}; \qquad \Delta = \frac{d^2}{dy^2} - \alpha^2; \quad c = \frac{\omega}{\alpha};$$

ψ is the stream function of the carrier medium ($u_{fx} = \psi'$, $u_{fy} = -i\alpha\psi$).

Let us consider a dispersed medium flow in a plane channel with solid walls at $y = \pm 1$ with the velocity profile $U = 1 - y^2$ (the transverse coordinate y is normalized by the channel halfwidth L). Two-dimentional disturbances in such a flow are subject to the usual nonpenetration and attachment conditions at the boundaries:

$$\psi(1) = 0, \qquad \psi'(1) = 0; \tag{5}$$

$$\psi(-1) = 0, \qquad \psi'(-1) = 0. \tag{6}$$

Since the problem is symmetric with respect to the $y = 0$ plane its solutions must have either a symmetric stream function

$$\psi'(0) = 0, \qquad \psi'''(0) = 0, \tag{7}$$

or antisymmetric one

$$\psi(0) = 0, \qquad \psi''(0) = 0. \tag{8}$$

In Poiseuille flows of homogenuous fluid only the symmetric mode is unstable. As has been mentioned already the flow stability can be reduced by the addition of small particles that are uniformly distributed over the flow space. Such stability reduction is caused by an increase in the medium's effective density, which is equivalent to a flow-velocity increase, and cannot generate a new unstable mode. With a nonuniform dispersed-phase distribution, the effective density of the medium becomes variable, and it is necessary to investigate both the symmetric disturbances subject to conditions (5) and (7) and antisymmetric ones subject to (5) and (8). Nonetheless, a comprehensive study of symmetric mode behavior is of value in its own right. It is clear that at sufficiently small particle concentrations the spectrum of the disturbances is close to that of a pure homogeneous fluid. For a mode that is stable in a pure fluid to become unstable in a two-phase one, if it is

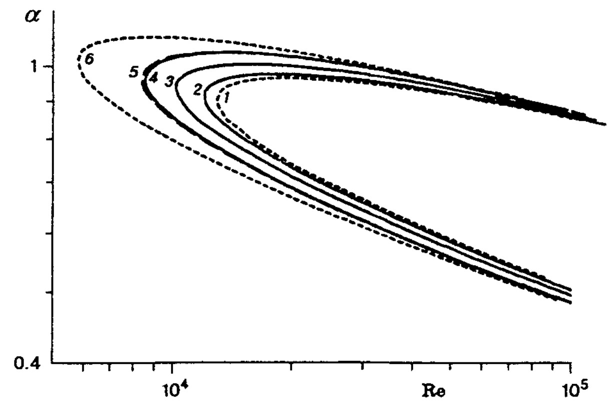

Fig. 1

possible at all, a critical particle concentration is necessary. So, a concentration interval always exists in which the flow stability is determined by the symmetric mode behavior.

Within the limits of the present work the problem (4), (5), and (7) was solved numerically using methods of orthogonalization and differential sweeping. Testing of the schemes used was performed on a problem of linear stability of a Poiseuille flow of a homogeneous fluid [2] and showed good agreement with the data of [5].

Results of Calculations. Let the dispersed-phase distribution be of the form

$$f(y) = f_V(\sigma)\exp(-y^2/\sigma^2) \qquad (-1 \leqslant y \leqslant 1)\,, \tag{9}$$

where the normalizing factor f_V is such that as σ changes the total number of particles in the channel is conserved. At the limit $\sigma \to \infty$ this distribution transforms into a uniform distribution, whereas at $\sigma \ll 1$ one gets a "thin dust layer" limit. In such a layer the mass particle concentration can be arbitrarily high, but the surface density

$$f_S = \int_{-1}^{1} f(y)dy = \int_{-1}^{1} f_V(\sigma)\exp(-y^2/\sigma^2)dy$$

is finite. An increase in mass concentration, however, does not mean an increase in the number of particles in the system. It is caused only by a decrease in dust-layer volume. Begining from some σ, the layer becomes arbitrarily thin, and in just this sense the mass concentration is arbitrarily large.

The choice of a dispersed-phase distribution in the form of (9) is justified for several reasons. First, the problem is of interest from the point of view of flow-stability control. Obviously the generation of a more or less thin dust layer is most economically and technically feasible. Second, any alternate distribution of particles can be represented as a set of thin layers. And, finally, if the dust layer is thin enough, the precise form of $f(y)$ becomes unimportant, and only the surface density f_S is of importance.

In Fig. 1 curves of neutral stability for media with particle distributions given by (9) are shown for different values of dust-layer thickness σ. Here $S = 2.5 \cdot 10^{-4}$, $f_S = 0.1$. Curve 6 corresponds to a pure fluid, and curve 1 to a two-phase flow with a uniform particle distribution [$\sigma \to \infty$ in (9)]. Curves 2–5 correspond to $\sigma = 2$, 1, 0.5, and 0.2. As the dust layer thickness σ decreases the curves rapidly converge to a limit, which differs both from the case of a uniform distribution and from the case of a pure fluid. This limit is practically reached already at $\sigma = 0.5$, and so the flow stability is weakly affected by the redistribution of particles in the central part $|y| < 0.5$ of the flow.

The described character of flows with a particle-number density distribution of the form of (9) remains the same for a wide range of parameter f_S variation. The maximum stabilizing effect is reached with a uniform particle distribution inside the flow (the calculations presented in Fig. 1 were performed for a fixed number of particles in the flow space). A decrease in particle surface density will shift the neutral stability curves

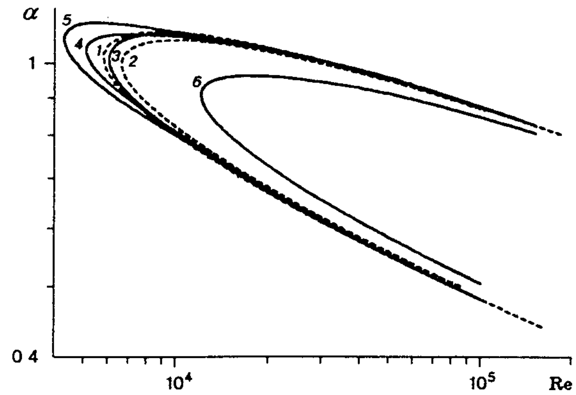

Fig. 2

obtained at the limit of a thin dust layer toward curve 6 in Fig. 1 of pure fluid neutral stability.

Let us now investigate the behavior of disturbances when the dust layer is shifted with respect to the flow axis. Since only symmetric disturbances are considered in the present work, we assume the presence of two dust layers located symmetrically with respect to the flow axis at distance ξ from it:

$$f(y) = f_V(\sigma, \xi)\frac{1}{2}\left[\exp(-(y-\xi)^2/\sigma^2) + \exp(-(y+\xi)^2/\sigma^2)\right] . \tag{10}$$

At $\xi = 0$, we have an axisymmetric dust layer (9); at $\sigma \ll \xi < 1$, two layers with surface particle density $f_S/2$.

Figure 2 shows how the flow stability changes with dust-layer displacement from the flow axis toward the boundary surfaces. The thickness of each layer is $\sigma = 0.1$, the total surface density of the particles is $f_S = 0.01$ (the surface density in each layer is $f_S/2 = 0.005$), and the relaxation time is $S = 2.5 \cdot 10^{-4}$. Curves of the neutral stability of a pure fluid 1 and of a uniform dispersed medium 2 ($f = 0.01$, $S = 2.5 \cdot 10^{-4}$) are shown for comparison. The flow stability is not significantly affected by particle density redistribution in the central part of the flow $\xi = 0$–0.5 (the curves of neutral stability for $\xi = 0$, 0.1, 0.2, 0.3, 0.4, and 0.5 coincide within the accuracy of graphical plotting and are given in Fig. 2 by curve 3). With further shifting of the layers toward the boundaries, the stability initially decreases (curve 4 for $\xi = 0.72$), reaches a minimum at $\xi = 0.77$ (curve 5), and then begins to grow again as the dust layers approach the boundaries (curve 6 for $\xi = 1.0$).

Such flow behavior is typical. Variation of the dust layer thicknesses changes the results only quantitatively. In Fig. 3, profiles of the critical Reynolds number $\mathrm{Re_c}(\xi)$ are given for flows with dust layers (10) of different thicknesses σ. The particle density in each layer is $f_S/2 = 0.005$; the relaxation time is $S = 2.5 \cdot 10^{-4}$. Curves 1–3 correspond to dust-layer thicknesses $\sigma = 0.1$, 0.05, and 0.02, respectively. It can be seen that the maximal stability is reached not when the layer is located immediately near the wall, but at some intermediate position $\xi \sim 0.95$. In addition, one more local maximum exists at $\xi \sim 0.75$. In the case of a thick layer, these effects are "smoothed." The described dependence of $\mathrm{Re_c}$ on ξ is due to a sharp change in the disturbance phase velocity, which happens in the vicinity of the critical layer and inside it. The dependence of the disturbance phase velocity c on parameter ξ at different values of σ is given in Fig. 4; curves 1–3 correspond to $\sigma = 0.1$, 0.05, and 0.02.

An important factor which determines flow stability is the dust-layer relaxation time. The dependence $\mathrm{Re_c}(\xi)$ is presented in Fig. 5 for different values of the medium's relaxation time S. Here $\sigma = 0.05$, $f_S = 0.01$. Curves 1-5 correspond to relaxation times $S = 4 \cdot 10^{-3}$, 10^{-3}, $2.5 \cdot 10^{-4}$, 10^{-4}, and 10^{-6}. It is seen that the greatest impact on flow stability is produced by small particles. Since at $S\mathrm{Re} \ll 1$ the medium behaves as a single phase medium with enhanced density, it can be assumed that the main effect of a dust layer on flow stability at small S is provided by the local change of medium's effective density in the vicinity of the critical layer. Such a medium can be described in the context of single-component hydromechanics; the appropriate

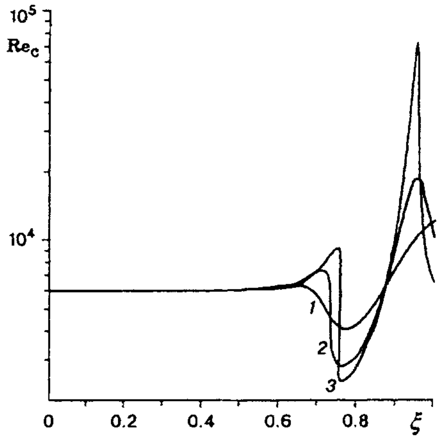

Fig. 3

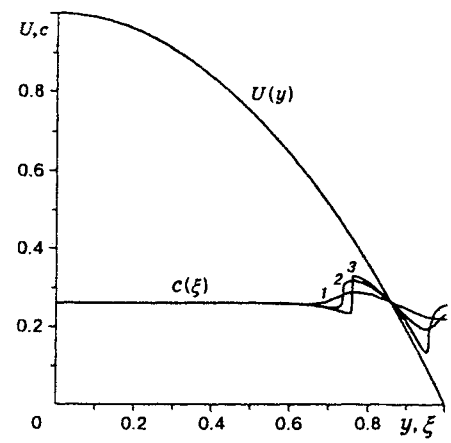

Fig. 4

equations are

$$\nabla \cdot \mathbf{V} = 0\,, \qquad \rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}\right) = -\nabla P + \frac{1}{\mathrm{Re}}\Delta\mathbf{V}\,,$$

where $\rho(y)$ is the medium's density; $\mathbf{V}$ is its velocity field. A standard derivation leads to the stability equation

$$\rho[(U-c)\Delta\psi - U''\psi] + \rho'[(U-c)\psi' - U'\psi] = \frac{1}{i\alpha\mathrm{Re}}\Delta^2\psi\,, \tag{11}$$

which can also be obtained directly from (4) if one puts $S = 0$, $\rho(y) = 1 + f(y)$. As a result of solution of Eq. (11) with boundary conditions (5) and (7), the dependence $\mathrm{Re_c}(\xi)$ was found for a medium with a variable density profile $\rho(y)$, which coincided with curve 5 in Fig. 5 ($S = 10^{-6}$) with graphical accuracy, proving the above assumption. As S increases phase "decoupling" takes place, with the result that the effective density of the medium in the dust layer decreases, and, as a consequence, its effect on flow stability is reduced.

As the particle concentration in each layer increases, the character of stability changes. In Fig. 6, curves of neutral stability are given for $f_S = 0.1$, $S = 2.5\cdot 10^{-4}$, $\sigma = 0.1$. As with smaller densities f_S, particle redistribution in the central flow area weakly influences flow stability (curve 1 in Fig. 6 corresponds to $\xi = 0$–0.5). However, as the dust layer approaches criticality at relatively small flow velocities ($\mathrm{Re} < 1000$) along with the main area of instability (curve 2, which coincides with curve 1 within the accuracy of the points plotted) a new instability subdomain appears for the same disturbance mode (curve 2a for $\xi = 0.58$). The sizes of this subdomain increase rapidly (curves 3a and 3 at $\xi = 0.7$), and finally it merges with the main instability domain (curve 4 for $\xi = 0.75$). With further shifting of the dust layer toward the flow boundary its stability again increases up to $\mathrm{Re_c} > 400\,000$ (curve 5 corresponds to $\xi = 0.887$, and curve 6 corresponds to $\xi = 1.0$).

Analysis of Disturbance Energy. The rate of two-phase-medium energy increase is determined by the relation

$$\frac{dE}{dt} = \frac{d}{dt}(E_f + E_p) = \frac{d}{dt}\frac{1}{2}\int_\Omega dy(v_f^2 + \rho_p v_p^2)\,,$$

where Ω is the flow space. The terms of this expression are

$$\frac{dE_f}{dt} = \int_\Omega dy\, \mathbf{v}_f \cdot \left(\frac{\partial \mathbf{v}_f}{\partial t} + \mathbf{V}_f\cdot\nabla\mathbf{v}_f\right)\,,$$

$$\frac{dE_p}{dt} = \int_\Omega dy\, \rho_p\mathbf{v}_p \cdot \left(\frac{\partial \mathbf{v}_p}{\partial t} + \mathbf{V}_p\cdot\nabla\mathbf{v}_p\right) + \int_\Omega dy\, \frac{v_p^2}{2}\left(\frac{\partial \rho_p}{\partial t} + \mathbf{V}_p\cdot\nabla\rho_p\right)\,.$$

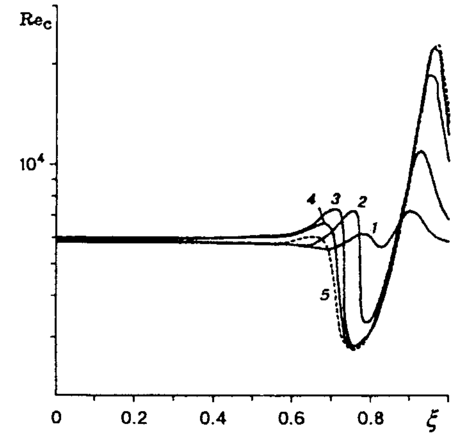

Fig. 5

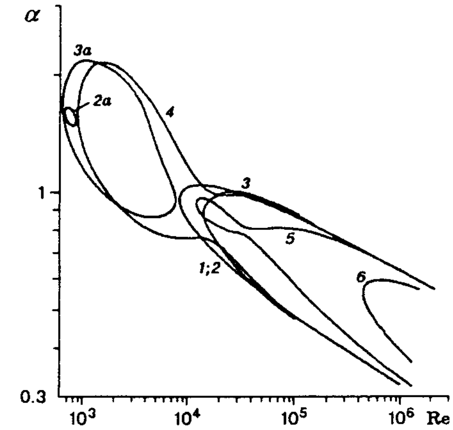

Fig. 6

Substituting functions (2) and taking into account the smallness of the disturbance amplitude, we obtain

$$\frac{dE}{dt} = \int_\Omega dy\, \mathbf{v}_f \cdot \left(\frac{\partial \mathbf{v}_f}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{v}_f\right) + \int_\Omega dy f \mathbf{v}_p \cdot \left(\frac{\partial \mathbf{v}_p}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{v}_p\right). \tag{12}$$

Using linearized Eqs. (1) and (3) to simplify formula (12), and taking into account that on the Ω domain boundary the condition $\mathbf{v}_f = 0$ must be satisfied, we find

$$\frac{dE}{dt} = -\int_\Omega dy \tau_f U' - \frac{1}{\mathrm{Re}}\int_\Omega dy (\nabla\mathbf{v}_f):(\nabla\mathbf{v}_f) - \int_\Omega dy f \tau_p U' - \frac{1}{S\mathrm{Re}}\int_\Omega dy\, f(\mathbf{v}_f - \mathbf{v}_p)^2, \quad \tau_i = \langle v_{ix} v_{iy}\rangle \quad (i = f, p). \tag{13}$$

Here τ_f and τ_p are the Reynolds stresses of the carrier and dispersed phases, respectively.

In a single-phase fluid, the disturbance energy increase is associated with the first two terms of (13). The first corresponds to transformation of the main flow energy into disturbance energy and is most significant in the vicinity of the critical layer. The second term is quadratic and describes the viscous dissipation of disturbance energy near the flow boundary. The third and fourth terms in (13) are associated with the work of the interphase force; the third indicates that the particle is involved in the mechanism of disturbance interaction with the main flow, whereas the fourth is again of quadratic form and is governed by the viscous energy dissipation with mutual phase motion.

To analyze this equation in the general case, for arbitrary S and f, is not a simple task. We shall restrict our analysis of (13) to the special case of $S\mathrm{Re} \ll 1$. Using the interrelations (3) between the velocity fields of the particles $\mathbf{v}_p$ and carrier fluid $\mathbf{v}_f$ (3) it is easy to ascertain that with linear in $S\mathrm{Re}$ accuracy we have $\tau_f = \tau_p$, whereas $\mathbf{v}_p - \mathbf{v}_f = 0$. Taking this into account, we find

$$\frac{dE}{dt} = -\int_\Omega dy\, (1+f)\tau_f U' - \frac{1}{\mathrm{Re}}\int_\Omega dy\, (\nabla\mathbf{v}_f):(\nabla\mathbf{v}_f). \tag{14}$$

The last term in this expression is always negative, and the considered flow can be unstable if

$$\frac{dE}{dt} > 0 \quad \text{or} \quad -\int_\Omega dy\, (1+f)\tau_f U' > \frac{1}{\mathrm{Re}}\int_\Omega dy\, (\nabla\mathbf{v}_f):(\nabla\mathbf{v}_f). \tag{15}$$

If the particles in the flow are distributed uniformly, then f = const, whereas τ_f practically coincides with the corresponding expression for a homogeneous fluid τ_{f0} [the latter is easy to verify by analyzing Eq. (4), for instance]. The value of τ_{f0} in Poiseuille flow differs appreciably from zero only in the vicinity of the critical layer. Since in a homogeneous fluid there exists a Reynolds number range for which Poiseuille flow is stable, it is clear that the same two-phase fluid flow with a uniform particle distribution and with $S\mathrm{Re} \ll 1$ must be stable in some Reynolds number value region as well. Since the first term in expression (14) for a two-phase fluid is $1 + f$ times greater than in a homogeneous fluid, the two-phase flow destabilizes at smaller Reynolds numbers than the homogeneous fluid. From (15) follows a rough estimate of the relation between the critical Reynolds numbers of a homogeneous fluid Re_{c0} and of a two-phase one Re_c:

$$\mathrm{Re}_c = \mathrm{Re}_{c0}/(1+f).$$

With a nonuniform particle distribution in the flow, the character of the behavior of the Reynolds stresses of the carrier fluid τ_f can differ considerably from that of τ_{f0}. This can be seen already from analysis of the stream function Eq. (4), which for the situation under consideration reduces to (11), with $\rho = 1+f$. This equation differs from the conventional Orr–Sommerfeld equation by the factor $(1+f)$ in the first term on the left side and by the presence of the second term. For distribution (10), the function f' (as well as ρ') changes its sign in the vicinity of the point $y = \xi$. The absolute value of this function is small almost everywhere, and the second term in (11) can be neglected, unless ξ is in the vicinity of the critical layer. Inside the critical layer the stream function varies drastically and inclusion of this term is necessary. The sign alternation transforms it in the vicinity of the critical layer at $y > \xi$ firstly into an additional (to the viscous one) dissipative force, which leads to flow stabilization, as opposed to the homogeneous fluid case, and then into an active force, which leads to an increase in Reynolds stresses and, as a consequence, to instability development in precritical regimes of homogeneous fluid flow. Such behavior was observed in numerical calculations, the results of which were described in the above section.

The qualitative analysis conducted is valid at $S\mathrm{Re} \ll 1$; nevertheless, the described mechanisms of flow instability development and flow stabilization hold also at other values of the parameter $S\mathrm{Re}$. However, at $S\mathrm{Re} \gtrsim 1$ the force of the interphase interaction also becomes important. Let us prove this for the case of sufficiently thin layers, the surface particle concentration of which is comparatively small. In this case, distribution (10) can be replaced by

$$f(y) = \frac{1}{2} f_S[\delta(y-\xi) + \delta(y+\xi)] = f_S \delta_\xi(y) \qquad (f_S \ll 1)$$

and the parameters of the disturbed flow can be considered close to those of a homogeneous fluid ω_0, $\mathbf{v}_{f0}$, τ_{f0}, and E_0:

$$\omega = \omega_0 + f_S\omega_1, \quad \mathbf{v}_f = \mathbf{v}_{f0} + f_S\mathbf{v}_{f1}, \quad \tau_f = \tau_{f0} + f_S\tau_{f1}, \quad \frac{dE}{dt} = \frac{dE_0}{dt} + f_S\frac{dE_1}{dt} = f_S\frac{dE_1}{dt}. \tag{16}$$

Here the following relation, which is valid for neutral disturbancies of $\mathbf{v}_{f0}$ in a homogeneous fluid, was used

$$\frac{dE_0}{dt} = -\int_\Omega dy\, \tau_{f0}U' - \frac{1}{\mathrm{Re}}\int_\Omega dy\, (\nabla\mathbf{v}_{f0}) : (\nabla\mathbf{v}_{f0}) = 0.$$

Substituting functions (16) into Eq. (13), we obtain

$$\frac{dE_1}{dt} = -\int_\Omega dy\, \tau_{f1}(y)U'(y) - \frac{2}{\mathrm{Re}}\int_\Omega dy\, (\nabla\mathbf{v}_{f0}) : (\nabla\delta_\xi\mathbf{v}_{f1}) - \tau_{p0}(\xi)U'(\xi) - \frac{1}{S\mathrm{Re}}(\mathbf{v}_{f0}(\xi) - \mathbf{v}_{p0}(\xi))^2. \tag{17}$$

The sign of dE_1/dt determines whether there is an increase or decrease of flow stability with particles admixing. The two first terms in (17) are associated with the change in the disturbance eigenfunction caused by a change of the medium's effective density. The last two terms in (17) describe the work produced by the interphase force. They can be determined by (3) if the disturbance function $\mathbf{v}_{f0}$ of a homogeneous fluid is given. Let us combine the terms pairwise according to their meanings: $dE_1/dt = e_\rho(\xi) + e_f(\xi)$.

In Fig. 7 the dependences $dE_1/dt(\xi)$ and $e_f(\xi)$ are presented for the cases of coarsely and finely dispersed media. It is seen that for the finely dispersed medium ($S = 10^{-6}$) the main contribution to dE_1/dt (curve 1) comes not from e_f (curve 2), but from e_ρ, that is, the change in the medium's density profile in the vicinity of the critical layer is of controlling importance for flow stability. For the coarsely dispersed medium the terms e_ρ and e_f are comparable in order of magnitude (curves 3 and 4 for dE_1/dt and e_f, respectively, for $S = 10^{-2}$).

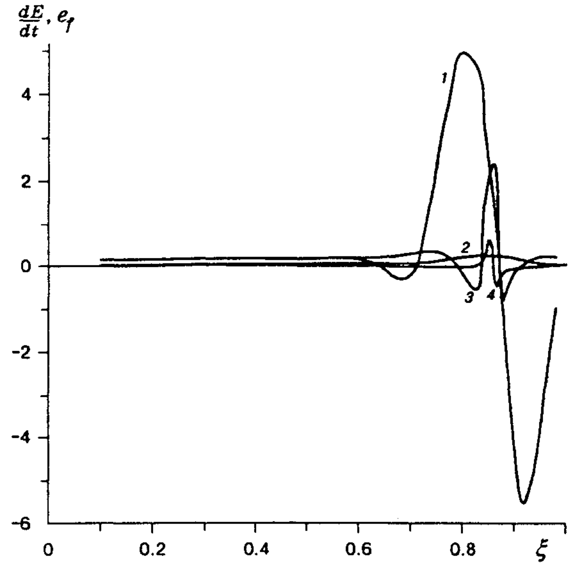

Fig. 7

Conclusions. The calculations which were reviewed in the preceding sections show that the stability properties of the Poiseuille flow of a two-phase medium are drastically changed by the addition of particles to the flow. These changes depend greatly on whether the particle distribution is uniform or not. If the particles are large enough, the degree of their distribution nonuniformity will affect the character of instability development much more weakly than in the fine-particle case. It is important to emphasize that the same particles can, depending on conditions (profiles of the distribution, density, flow velocity etc.), stabilize as well as destabilize the flow. This enables one to use particles for effective control of the development of flow instability. In the present work only the behavior of a symmetric mode of disturbances was studied. The investigations carried out with the author's participation reveal that under certain conditions and with sufficient particle concentration values, at least another [antisymmetric (5) and (8)] disturbance mode is destabilized. A detailed description of the changes in the disturbance spectra, however, is beyond the scope of the present work.

The great variety of possible dispersed-phase profiles $f(y)$ does not permit one to conduct an exhaustive investigation of the effect of an arbitrary particle distribution on flow stability. However, as has been pointed out already, any dispersed-phase distribution can be considered a superposition of a set of dust layers. Therefore a qualitative view of flow stability with other particle-concentration profiles is available based on the results of the present work. The change in the structure of the disturbance equations in the case of a nonuniform distribution is of principal importance. An additional term appears in the equation, which is absent in the case of a uniform particle distribution. This changes considerably the conventional stability criteria. Consider, in particular, the inviscid limit of Eq. (4). We assume that with $\mathrm{Re} \to \infty$ simultaineously $S\mathrm{Re} \to 0$ (such a situation is quite possible and is realized, for instance, in large scale flows, because $\mathrm{Re} \sim L$, whereas $S \sim L^{-2}$). Then an equation for disturbances follows from (11):

$$\rho\psi'' + \rho'\psi' - \rho\alpha^2\psi - \frac{(\rho U')'}{(U-c)}\psi = 0. \tag{18}$$

As is known [6], solutions of the Rayleigh equation obtained by integration along the real axis coincide with the limiting solutions of the Orr–Sommerfeld equation only in the case of increasing disturbances. It can be shown, without resorting to a complicated analysis, that Eq. (18) has the same properties with respect to "viscous" Eq. (11). Actually, the solution of the Rayleigh equation in a complex plane is not unique, because of the existence of a singular point y_c, which is determined by the condition $U(y_c) = c$, where the U is an analytical continuation of the velocity profile function into the complex plane. Therefore, in the complex plane

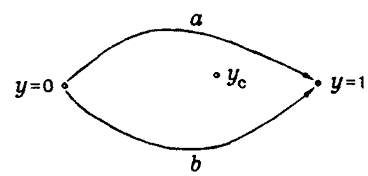

Fig. 8

two classes of integration paths exist, which differ in the way in which the singular point y_c is handled (the paths are denoted by (a) and (b) in Fig. 8). The solution of the viscous problem along one of the paths does not coincide with the solutions along the other one. The true path is determined by an analysis of the limiting behavior of the solution of the Orr–Sommerfeld equation for a negligibly small viscosity (Re $\to \infty$). Such an analysis was conducted in [6] and, in particular, came to the already mentioned conclusion: integration along the real axis can be used only for increasing disturbances. But, as is easily seen, the critical points of Eq. (18) coincide with those of the Rayleigh equation. Besides, there is a continuous passage to the limit $\rho(y) \to 1$ that transforms (11) into an Orr–Sommerfeld equation but (18) into a Rayleigh equation. Since the location of the critical point y_c remains the same, paths (a) and (b) transform into one another. Hence, in the integration of (18), the integration path must be chosen by the same rules as in integration of the Rayleigh equation.

Let a growing solution of (18) exist: $\operatorname{Im} c > 0$. We multiply this equation by the function ψ^*, which is the complex conjugate of ψ, and substract from the resulting equation its complex conjugate. As a result we have

$$\rho(\psi''\psi^* - \psi^{*\prime\prime}\psi) + \rho'(\psi'\psi^* - \psi^{*\prime}\psi) = \frac{c - c^*}{|U - c|^2}(\rho U')'|\psi|^2 .$$

It is apparent, that both sides of this equation are purely imaginary. If one introduces the real quantity

$$G = -i/2\rho(\psi'\psi^* - \psi^{*\prime}\psi) ,$$

the latter equation can be written as

$$\frac{d}{dy}G = \frac{\operatorname{Im} c}{|U - c|^2}(\rho U')'|\psi|^2 .$$

On the boundaries of the domain the condition $\psi = 0$ is satisfied, with the result that G must vanish. It follows clearly that the value dG/dy must reverse its sign in the interval between the flow boundaries. Hence, somewhere in this interval the condition $(\rho U')' = 0$ is satisfied. Thus we obtain a new necessary condition for the existence of growing disturbances. This condition is a generalization of the Rayleigh theorem about the point of inflection for media with a variable density profile.

The above reasoning forces one to suppose that the spectra of disturbances in heterogeneous media flows can differ qualitatively from those of a homogeneous flow. The authors intend to devote a separate work to study these spectra and, in particular, to investigate the behavior of other modes of instability.

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